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Design-Oriented Structural-Model Order Reduction of Strain-Actuated Flight-Vehicle Structures

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This paper assesses the accuracy of a number of structural-model order-reduction techniques as applied to strain-actuated ("smart") flight-vehicle structures modeled with finite elements (FE). Model order reduction is an essential facet when working in a design-oriented optimization environment. Therefore, to eventually build a design-oriented multidisciplinary optimization (MDO) capability for large-scale strain-actuated structures (e.g., aircraft, spacecraft), insight into the affects of model order reduction on the accuracy of design constraints and sensitivities must first be gained. In this paper reduced-order models are solved using mode displacement (MD) and mode acceleration (MA) methods. Furthermore, the effect of the chosen reduction bases on model accuracy is also investigated. Specifically, the total number of basis modes used is varied. Fictitious masses (FMs) may be added to areas of the structure where local effects predominate, and mass-normalized static solution vectors (Ritz vectors) of important actuation modes may be appended to the bases. By examining the convergence of design constraints and their sensitivities for a variety of model order-reduction approaches (solution methods and reduction bases), valuable insight is gained into the technical challenges that must be addressed when creating a robust MDO capability for strain-actuated structures.

Background and Motivation

DEVELOPMENT of new actuation and active-structures technologies[1–5] has reached a stage where serious studies of the effects of such technology on vehicle design can begin. In 20 years or so of intense research activity in the area known as "smart structures," new types of actuators and active materials were developed and studied. Basic research on structural concepts, where such materials and actuators are utilized, was also carried out. Concepts for a new generation of airplanes—"morphing airplanes"—began to take shape.[6–11] Such vehicles, according to recent research and development drives and associated publications, will be capable of adaptation and shape variation during flight to an extent beyond what designs of the past could offer.

Theory and computational analysis of aerospace-oriented active materials and new reduced-sized fast actuators were developed and correlated with experimental results using beams, plates, and prototype structural components (e.g., wings, control surfaces, and engine inlets). In most, if not all, of the design studies, considerable simplifications were made in the computational structural models.[12–16] Although beam and plate (as well as equivalent-beam and equivalent-plate) models of airplane structures are valuable in basic research,[13–27] they lack the generality of finite-element models, and they have serious limitations, especially when local effects become important.[28–30] Such local effects include concentrated forces, structural discontinuities, and local-action actuators. Local effects are also important when accurate stress predictions throughout a structure are sought, and, in optimization, when sensitivities of structural and aeroelastic response to changes in design variables are required.

If the potential of morphing vehicles is to be assessed, studies must be based on rigorous design optimization taking into account all important design constraints and criteria. Furthermore, these studies must include computational modeling that is capable of capturing the local and global behavior of real strain-actuated structures. Finite-element analysis (FEA) of thermally excited structures has long been available in commercial codes. Extensions to strain actuation using piezoelectric materials and shape-memory alloys were introduced more recently and have been used in some design studies of aerospace structures.[31–37] Most of those studies, however, involve either large general-purpose commercial codes, ad hoc modification of existing codes, or special-purpose modules interacting with general-purpose codes. More important, consideration of model order-reduction issues, as applied to strain-actuated structures, has been lacking in these research endeavors.

In this paper a dedicated design-oriented structural-modeling capability for strain-actuated flight vehicles is used to investigate model order-reduction issues. Model order reduction accelerates computation times and reduces computational resources. It is a critical requirement for any optimization framework used to solve large-scale problems. However, model order reduction is problematic in cases where local effects in the structure are important because the process may "smear" local effects in a global reduced-order approximation. As noted, strain-actuated structures involve local effects and these effects must be included in the optimization problem if realistic designs are sought. With careful consideration given to the reduced-order solution methods and the reduction basis chosen, the "smearing" of local effects may be minimized. This paper highlights how this is achieved using some simple structural-response problems. Direct and adjoint solution approaches are derived herein. The direct approach is the most common and well understood. The adjoint approach is interesting, however, because it gives additional insight into the local nature of the problem at hand.

Structural Dynamic Responses via Direct and Adjoint Solutions

The Fourier-transformed equations of motion for a linear elastic structure subject to the action of external forces F and control inputs v are

$$(-\omega^2[M] + (1 + jg)[K])\{u(j\omega)\} = \{F(j\omega)\} + [B]\{v(j\omega)\} \quad (1)$$

The mass and stiffness matrices are M and K respectively. Hysteretic structural damping is assumed. The matrix B relates control inputs to actuator loads imposed upon the structure at associated nodes. The frequency-dependent unknown motion vector is $u(j\omega)$.

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Individual displacements, strains, or stresses in elements can be obtained from a response equation of the type

$$y(j\omega) = \{c\}^T\{u(j\omega)\} \tag{2}$$

where the output $y(j\omega)$ can be displacement, rotation, strain, or stress, as determined by the vector $\boldsymbol{c}$ (real) used.

The sensitivity of displacements with respect to some structural-design variable can be obtained by implicit differentiation of Eq. (1)[38]:

$$(-\omega^2[M] + (1+jg)[K])\frac{\partial\{u(j\omega)\}}{\partial DV} = -\left(-\omega^2\frac{\partial[M]}{\partial DV} + (1+jg)\frac{\partial[K]}{\partial DV}\right)\{u(j\omega)\} + \frac{\partial[B]}{\partial DV}\{v(j\omega)\} \tag{3}$$

It is assumed in the derivation leading to Eq. 3 that the external forces and input voltages do not change with changes in structural-design variables. In case sensitivities with respect to external forces or input voltages are required, derivation of the proper sensitivity equations is straightforward.

Both Eqs. (1) and (3) can be written in the form

$$[A(j\omega)]\{x(j\omega)\} = \{b(j\omega)\} \tag{4}$$

where

$$[A(j\omega)] = (-\omega^2[M] + (1+jg)[K]) \tag{5}$$

The analysis solution $u(j\omega)$ and the sensitivity solution $\partial[u(j\omega)]/\partial DV$ are $x(j\omega)$ solutions corresponding to different b right-hand sides:

$$\{b(j\omega)\} = \{F(j\omega)\} + [B]\{v(j\omega)\} \tag{6}$$

and

$$\{b(j\omega)\} = -\left(-\omega^2\frac{\partial[M]}{\partial DV} + (1+jg)\frac{\partial[K]}{\partial DV}\right)\{u(j\omega)\} + \frac{\partial[B]}{\partial DV}\{v(j\omega)\} \tag{7}$$

respectively.

The response function $y(j\omega)$ can also be obtained from a solution of an adjoint problem. Recognizing [Eqs. (1), (2), and (4)] that

$$y(j\omega) = \{\boldsymbol{c}\}^T\{x(j\omega)\} = \{c\}^T[A(j\omega)]^{-1}\{b(j\omega)\} \tag{8}$$

an adjoint solution is defined:

$$\{\boldsymbol{\eta}\}^* = \{\boldsymbol{c}\}^T[A(j\omega)]^{-1} \rightarrow [A(j\omega)]^*\{\eta\} = \{c\} \tag{9}$$

The $*$ superscript denotes adjoint vectors and matrices (transposed and complex conjugate).

The vector $\boldsymbol{c}$ is real, and so its adjoint and transpose are identical. The response y (displacement, strain, stress, or their sensitivities with respect to design variables) can be obtained by solving for the adjoints [Eq. (9)] and using Eq. (8):

$$y(j\omega) = \{\boldsymbol{\eta}\}^*\{b(j\omega)\} \tag{10}$$

In the frequency response case,

$$[A(j\omega)]^* = (-\omega^2[M] + (1-jg)[K]) \tag{11}$$

because of the symmetry of the mass and stiffness matrices of a structural dynamic system.

The availability of the adjoint solution offers some interesting insights. By combining Eqs. (9) and (10) we can write the equation for adjoint solutions in the form

$$(-\omega^2[M] + (1-jg)[K])\{\boldsymbol{\eta}(j\omega)\} = \{\boldsymbol{c}\} \tag{12}$$

This is an equation for a structural-dynamic system with negative hysteretic damping and an external loading $\boldsymbol{c}$. It is interesting to examine the nature of this external loading in cases involving different responses $y(j\omega)$. When the response is stress in a particular element, the $\boldsymbol{c}$ vector contain nonzero entries only at the degrees of freedom corresponding to the nodes of that element. Moreover, these entries have both positive and negative values and represent a local system of forces acting in different directions on the element.

In the case of a unidirectional truss element, for example, connecting degrees of freedom k and l, the $\boldsymbol{c}$ vector contain nonzero entries at the k and l location only:

$$\{\boldsymbol{c}\}^T = \{0 \quad 0 \quad -1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0\}$$
$$\qquad \ldots k \ldots \qquad \ldots l \ldots$$

This represents in the adjoint problem two (self-equilibrating) forces acting on the nodes of the element in opposite directions.

Examination of the direct-sensitivity problem reveals a similar effect [Eqs. (3), (5), and (7)]. The right-hand side of the sensitivity problem for displacements is given by Eq. (7):

$$\{b(j\omega)\} = -\left(-\omega^2\frac{\partial[M]}{\partial DV} + (1+jg)\frac{\partial[K]}{\partial DV}\right)\{u(j\omega)\} + \frac{\partial[B]}{\partial DV}\{v(j\omega)\}$$

When a design variable associated with a single finite element is considered (say, the thickness of composite layers, the area of a spar or rib cap, the thickness or cross-sectional area of a strain actuator), then the matrices

$$\frac{\partial[M]}{\partial DV}, \quad \frac{\partial[K]}{\partial DV}, \quad \frac{\partial[B]}{\partial DV}$$

contain nonzero entries corresponding only to the degrees of freedom at the nodes of that element. Again both positive and negative matrix terms will be involved. In the case of a truss member's stiffness matrix, for example, when the design variable is the cross-sectional area A, then

$$\left[\frac{\partial K}{\partial A}\right] = \frac{E}{L}\begin{bmatrix} 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ 0 & 1 & 0 & \vdots & -1 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & -1 & 0 & \vdots & 1 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \ldots k \\ \\ \\ \ldots l \\ \\ \\ \end{matrix}$$
$$\ldots k \ldots \qquad \ldots l \ldots$$

Multiplying by the vector of displacements $\boldsymbol{u}$ we obtain

$$\left[\frac{\partial K}{\partial A}\right]\boldsymbol{u} = \frac{E}{L}\begin{Bmatrix} 0 \\ u_k - u_l \\ 0 \\ \vdots \\ -(u_k - u_l) \\ 0 \\ 0 \end{Bmatrix} = \frac{E}{L}(u_k - u_l)\begin{Bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ -1 \\ 0 \\ 0 \end{Bmatrix}$$

The displacement-sensitivity problem, then, is also a problem of an equivalent structural-dynamic system subject to equal and opposing forces acting *locally* on the nodes of the element being changed. In a statically indeterminate structure activated by a single internal actuator connecting certain degrees of freedom, actuation creates a set of internal self-equilibrating forces acting inside the structure. This, again, is a local effect similar to the cases discussed previously.

The identification of cases in which a structure is loaded by a localized set of concentrated forces acting in opposite directions is important when model order reduction is considered, as is discussed in the next section.

Order Reduction

The most common method of order reduction in structural dynamics is the mode displacement method (MD),[39–41] where the displacement vector $\boldsymbol{u}$ is approximated using a superposition of some mode shapes constituting a reduced basis:

$$\{\boldsymbol{u}\} \approx [\{\phi_1\} \quad \{\phi_2\} \quad \ldots \quad \{\phi_n\}] \begin{Bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{Bmatrix} = [\Phi]\{q\} \tag{13}$$

The number of reduced-base vectors is much smaller than the number of degrees of freedom in the full-order model. The equations of motion (1) can now be reduced in order by substituting Eq. (13) and premultiplication by the transpose of the matrix Φ.

$$(-\omega^2[\Phi]^T[M][\Phi] + (1 + jg)[\Phi]^T[K][\Phi])\{q(j\omega)\}$$
$$= [\Phi]^T\{F(j\omega)\} + [\Phi]^T[B]\{v(j\omega)\} \tag{14}$$

It is assumed here that the matrix Φ is real, containing real base vectors ϕ_i. These can be obtained as 1) a subset of the low-frequency natural mode shapes of the structure, 2) mode shapes of a related structure loaded with large fictitious masses at key degrees of freedom,[41–43] 3) some other set of base vectors such as Ritz vectors,[44] or 4) a mix of natural modes and Ritz vectors.[45,46]

Sensitivity of the MD reduced-order response can be obtained by differentiation of Eqs. (14). When a set of *fixed*-basis vectors is used, the modal matrix Φ does not depend on design variables. Then

$$(-\omega^2[\Phi]^T[M][\Phi] + (1 + jg)[\Phi]^T[K][\Phi])\frac{\partial\{q(j\omega)\}}{\partial DV}$$
$$= \left(\omega^2[\Phi]^T\frac{\partial[M]}{\partial DV}[\Phi] - (1 + jg)[\Phi]^T\frac{\partial[K]}{\partial DV}[\Phi]\right)\{q(j\omega)\}$$
$$+ [\Phi]^T\frac{\partial[B]}{\partial DV}\{v(j\omega)\} \tag{15}$$

The reduced-order approximation of the response, when the direct method is used, is

$$y(j\omega) \approx \{\boldsymbol{c}\}^T[\Phi]\{q(j\omega)\} \tag{16}$$

For the sensitivities,

$$\frac{\partial y(j\omega)}{\partial DV} \approx \frac{\partial\{\boldsymbol{c}\}^T}{\partial DV}[\Phi]\{q(j\omega)\} + \{\boldsymbol{c}\}^T[\Phi]\frac{\partial\{q(j\omega)\}}{\partial DV} \tag{17}$$

It is well known that although the classical MD method (with natural modes) can be quite accurate if deformations are the response functions of interest, it converges slowly (with increased number of modes in the reduced base) when stresses or strains are considered.[39,40,47,48] The strain/stress convergence problem becomes usually more severe when the loading on the structure involves concentrated forces and moments. Convergence (with number of modes in the reduced base) is also slow when behavior sensitivity is considered with respect to the properties of a single element.[47,48]

A "smart" flight vehicle, controlled by strain actuators and other actuators located strategically in the structure, presents an order-reduction challenge when design optimization is involved. The actuators themselves introduce systems of local forces and moments. Sensitivities, as we have shown, can also be thought of as the results of systems of concentrated forces and moments acting locally. Natural low-frequency modes portray deformation of a global nature and cannot capture local effects well. "Smearing" of the input forces and truncation of the calculated responses are well-known problems in the MD method.[47,48] In addition, strains and stresses are themselves local in nature, and the convergence of displacement-formulation-based MD finite-element stress solutions is slower than displacement convergence. The convergence is even slower when stress and strain *sensitivities* are involved. Methods such as the fictitious mass (FM) method or Ritz vectors become less efficient.

Because of the possible presence of a large number of actuators, stress points, local design variables, temperature inputs, and so forth, reduced-order bases become large when enough base vectors are included to capture all possible local effects. If an efficient FE-based structural-modeling capability is sought for design optimization of actively controlled structures, special attention must be paid so that proper order reduction will be capable of capturing stress and strains and their sensitivities in the presence of point actuators and point loads. Such order reduction is required to reduce computational costs of the objective function and constraint evaluations, which have to be carried out repeatedly.

Mode Acceleration

Solution of MD reduced-order equations of motion usually yields accurate deformation results if enough mode shapes are included in the reduced basis. Stress recovery with MD order reduction suffers mainly because of truncation and smearing of stiffness effects.

In what is known as the mode acceleration (MA) method or the summation of forces method in aeroelasticity (SOF), the full-order equation of motion (1) is reorganized by moving the inertia terms to the right-hand side:

$$((1 + jg)[K])\{u(j\omega)\} = \omega^2[M]\{u(j\omega)\} + \{F(j\omega)\} + [B]\{v(j\omega)\} \tag{18}$$

First, a MD solution is calculated (Eq. 14). Then, the deformation vector on the right-hand side of the full-order equations is replaced by its MD approximant (see Appendix):

$$((1 + jg)[K])\{u(j\omega)\} = \omega^2[M][\Phi]\{q(j\omega)\}$$
$$+ \{F(j\omega)\} + [B]\{v(j\omega)\} \tag{19}$$

Thus, although an MD order reduction is considered sufficient to approximate accurately the distribution of inertial loads, the actual deformation (for stress-calculation purposes) under all loads is obtained from a full-order quasi-static solution using the full-order stiffness matrix. For the frequency-response case in Eq. (19), the left-hand-side coefficient matrix is independent of frequency. It can be LU decomposed once and then used repeatedly with different frequency-dependent right-hand sides.

Or, for a particular response [Eq. (2)],

$$y(j\omega) = \{\boldsymbol{c}\}^T\{u(j\omega)\} = \{\boldsymbol{c}\}^T[(1 + jg)[K]]^{-1}(\omega^2[M][\Phi]\{q(j\omega)\}$$
$$+ \{F(j\omega)\} + [B]\{v(j\omega)\}) \tag{20}$$

An adjoint solution of this problem is now defined:

$$\{\boldsymbol{\xi}\}^* = \{\boldsymbol{c}\}^T((1 + jg)[K])^{-1}, \qquad ((1 - jg)[K])^T\{\boldsymbol{\xi}\} = \{\boldsymbol{c}\} \tag{21}$$

Equation 20 is solved once for each response quantity needed and then

$$y(j\omega) = \{\boldsymbol{\xi}\}^*(\omega^2[M][\Phi]\{q(j\omega)\} + \{F(j\omega)\} + [B]\{v(j\omega)\}) \tag{22}$$

For sensitivities of response calculated by the MA method,

$$((1 + jg)[K])\frac{\partial\{u(j\omega)\}}{\partial DV} = -\left((1 + jg)\frac{\partial[K]}{\partial DV}\right)\{u(j\omega)\} + \omega^2[M][\Phi]$$
$$\times \frac{\partial\{q(j\omega)\}}{\partial DV} + \omega^2\frac{\partial[M]}{\partial DV}[\Phi]\{q(j\omega)\} + \frac{\partial[B]}{\partial DV}\{v(j\omega)\} \tag{23}$$

where u is the MA solution and q is the MD solution. Sensitivity of the response

$$\frac{\partial y(j\omega)}{\partial DV} \approx \frac{\partial\{c\}^T}{\partial DV}\{u(j\omega)\} + \{\boldsymbol{c}\}^T\frac{\partial\{u(j\omega)\}}{\partial DV} \tag{24}$$

With the adjoint formulation the sensitivities are obtained from

$$\frac{\partial y(j\omega)}{\partial DV} = \frac{\partial\{\xi\}^*}{\partial DV}(\omega^2[M][\Phi]\{q(j\omega)\} + \{F(j\omega)\} + [B]\{v(j\omega)\}) + \{\xi\}^*\left(\omega^2\frac{\partial[M]}{\partial DV}[\Phi]\{q(j\omega)\} + \omega^2[M][\Phi]\frac{\partial\{q(j\omega)\}}{\partial DV} + \frac{\partial[B]}{\partial DV}\{v(j\omega)\}\right) \tag{25}$$

The sensitivities $\partial\boldsymbol{\xi}/\partial DV$ are obtained by implicit differentiation of Eq. (21), in exactly the same way derivatives of static structural response are obtained.

Order-Reduction Studies

Design-oriented structural-dynamic modeling of strain-actuated structures, as we have discussed, has to accurately capture a number of important local effects, including the effect of concentrated forces, local strain inputs, stress, and stress sensitivities. This presents a major challenge when order-reduction techniques are used. The reduced bases used in such order-reduction methods have to be capable of capturing both global and local behavior of the structure when local inputs and behavior have to be evaluated at many locations.

To study such issues, a simple two-dimensional panel is modeled with finite elements in SMART,[49] the design-oriented FE modeling capability for strain-actuated structures developed at the University of Washington (Figs. 1 and 2), and reduced-order approximations using a number of methods are compared. In the following we examine 1) the standard MD method with subset of the natural modes

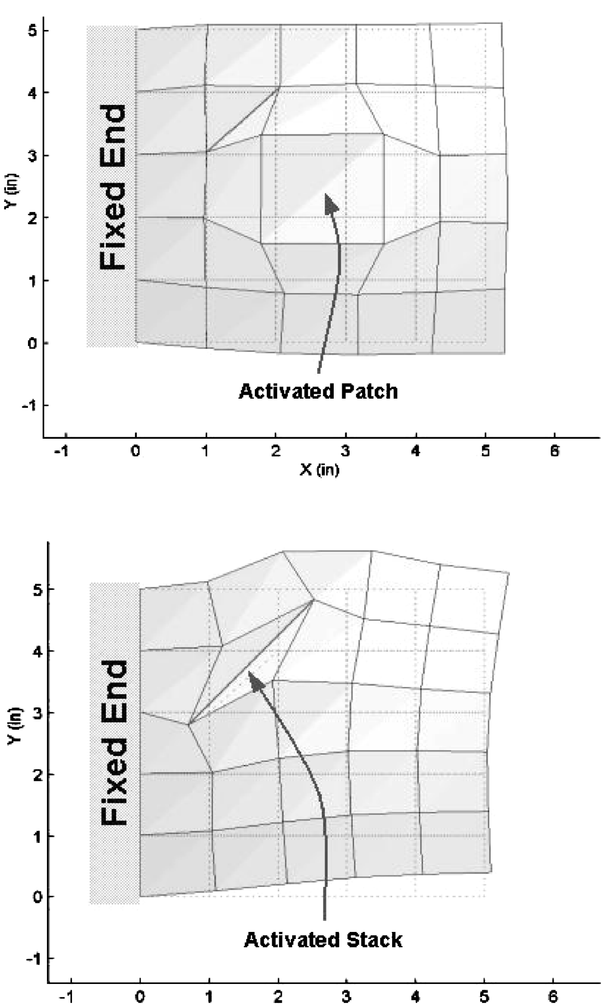

Fig. 1 Two-dimensional active panel: displacements due to patch and stack element activations.

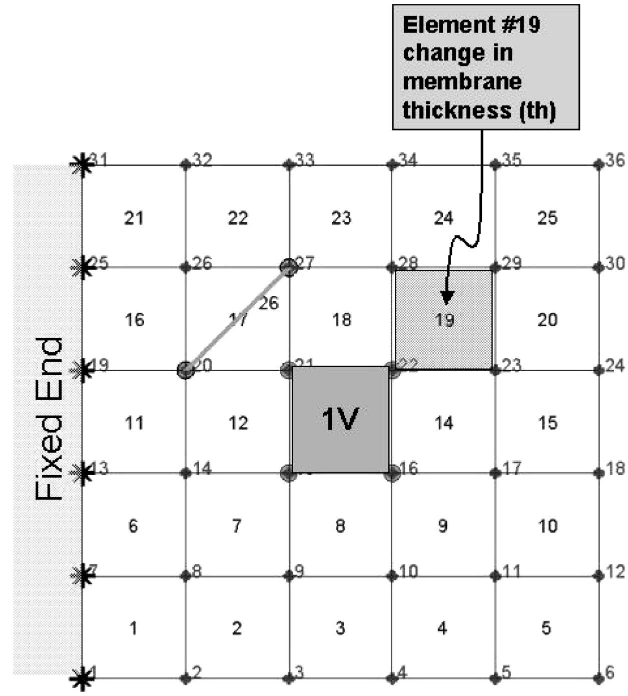

Fig. 2 Active patch element, variable-thickness element, and stress-response element on the two-dimensional panel.

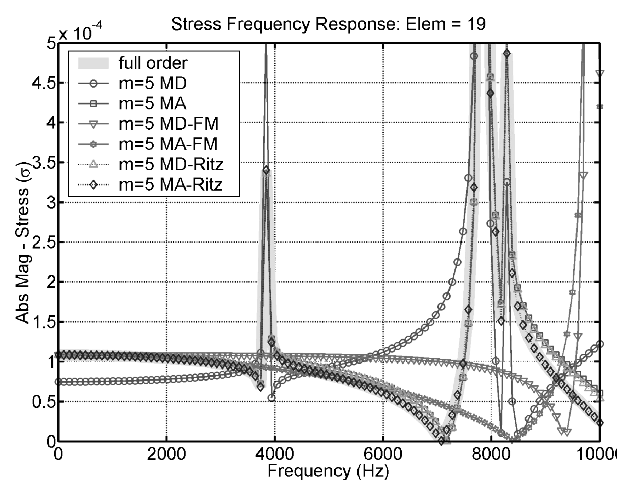

Fig. 3 STRESS response in element 19 using various reduced-order models with a reduced basis of dimension 5 (the response peaks are clipped for better resolution on the graph).

of the structure, 2) the MD method using FM modes,[50] where the FM matrix is proportional to stiffness matrices of active elements (MD-FM), and 3) a Ritz method, where a set of natural modes is augmented by a Ritz vector representing the deformation of the structure due to unit-actuator input (MD-Ritz). All these variants of MD are also used in an MA approach. The resulting cases are denoted MA, MA-FM, and MA-Ritz.

A few notes are in place regarding the FM and Ritz methods. FMs loaded at degrees of freedom where local effects are important lead to a structure whose mode shapes approximately span the space of the low-frequency modes shapes of the unaugmented structure and the space of Ritz vectors due to force inputs at the mass loaded degrees of freedom.[41,42] For a given size reduced base, any effort to capture local effects using FMs leads to a reduction of mode shapes available for capturing dynamic response at the upper end of the frequency range of interest. The same applies to the Ritz method used here, where Ritz vectors and natural mode shapes are combined to create a reduced base.[45,46]

Examinations of Figs. 3 and 4 show this effect. When only five vectors are allowed in the reduced base, both the FM and Ritz

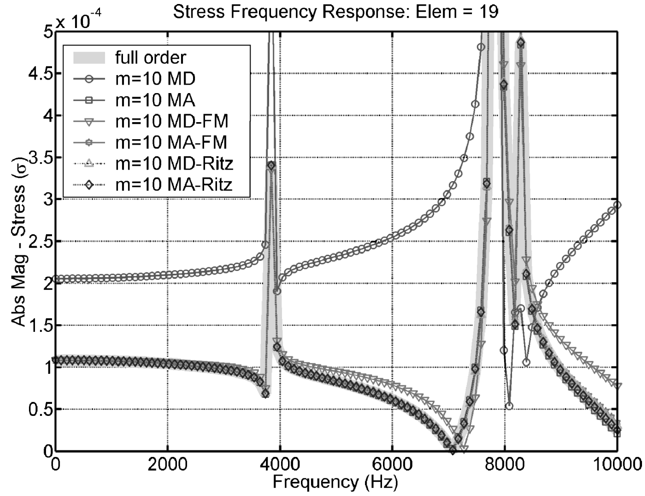

Fig. 4 STRESS response in element 19 using various reduced-order models with a reduced basis of dimension 10 (the response peaks are clipped for better resolution on the graph).

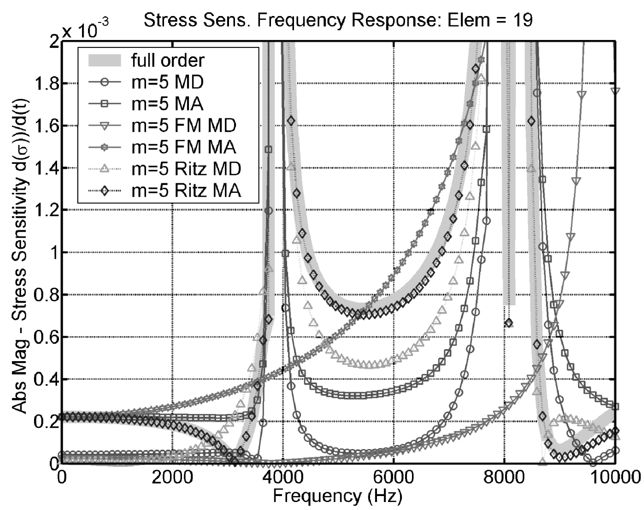

Fig. 5 STRESS SENSITIVITY response in element 19 using various reduced-order models with a reduced basis of dimension 5 (the sensitivity-response peaks are clipped for better resolution on the graph).

methods cannot capture dynamic behavior over the whole frequency band of interest. With 10 base vectors the accuracy improves and peaks of behavior at the high frequencies can now be captured. The classical MD method (based on natural modes) performs poorly when it comes to stresses, as shown here.

Reduced-order approximations for the stress *sensitivity* with respect to an element's thickness are shown in Figs. 5 and 6. With a local input load due to the actuator patch and a stress sensitivity response in one element with respect to thickness in another element are considered, the multitude of local actions becomes a challenge when order reduction is considered. Best performance, it seems, is obtained using an MA method with a reduced base of mixed natural modes and a Ritz vector of static response to actuation command.

A more realistic test problem is the high-altitude long-endurance (HALE) wing described in Ref. 23. This wing is modeled with finite elements in SMART and is shown in Fig. 7. Strain-activated layers are placed in the top and bottom skins of the wing. Figures 8 and 9 show convergence studies for approximate, reduced-order stress and stress-sensitivity responses on the high AR wing of Fig. 7. A strain input in one location, a change in thickness in another, and stress evaluated in yet another location are considered. The poor performance of classical MD (with natural modes) is evident. Special care must be taken to include enough vectors in the reduced base to allow sufficient accuracy of both stress and stress-sensitivity response.

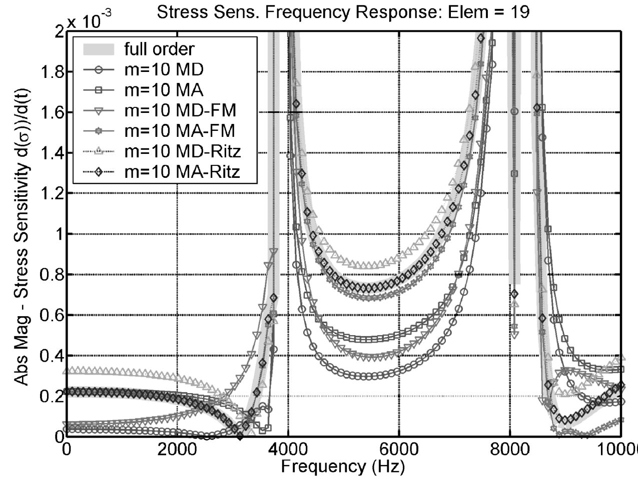

Fig. 6 STRESS SENSITIVITY response in element 19 using various reduced-order models with a reduced basis of dimension 10 (the sensitivity-response peaks are clipped for better resolution on the graph).

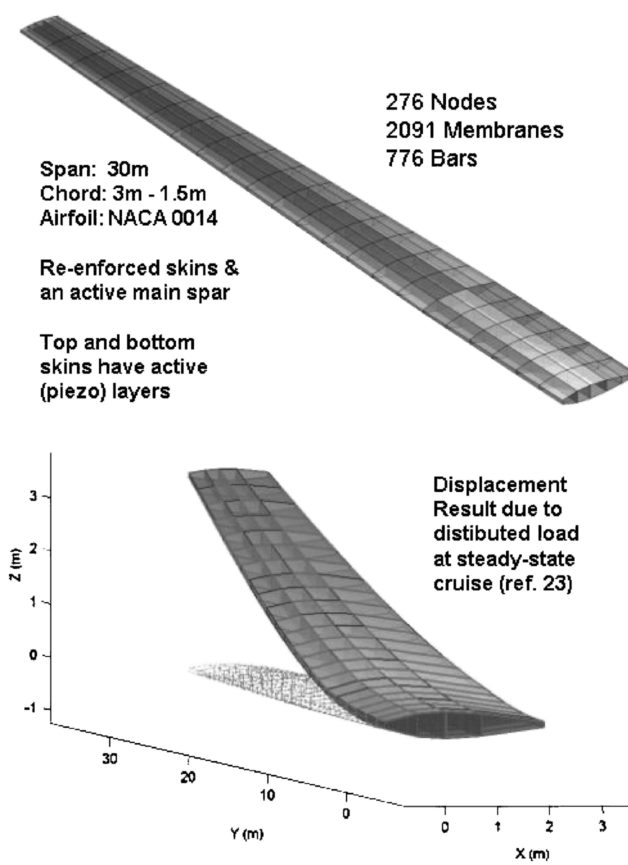

Fig. 7 High AR wing.

Results for the three MD and three MA variants at a frequency of 6 Hz are shown in Figs. 10 and 11.

It is clear that when the number of actuators increases so must the dimension of the reduced basis used for order reduction. For cases in which actuators respond to a single command to each actuator, and there is no change in the directional character of actuation strain during optimization (that is, induced actuator strain keeps the same ratio between its longitudinal and lateral components), augmentation of a natural-modes base by Ritz vectors describing responses to each actuator separately seem at this stage to offer a viable order-reduction approach. Additional studies of this problem are planned for the near future.

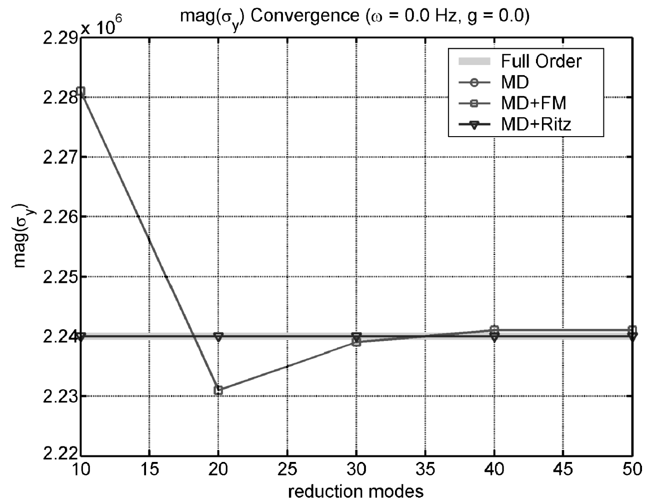

Fig. 8 High AR wing reduced-basis STRESS convergence studies (static case, no damping). (Note: Because of large errors the MD curve for the case of natural modes is out of the range shown in the figure.)

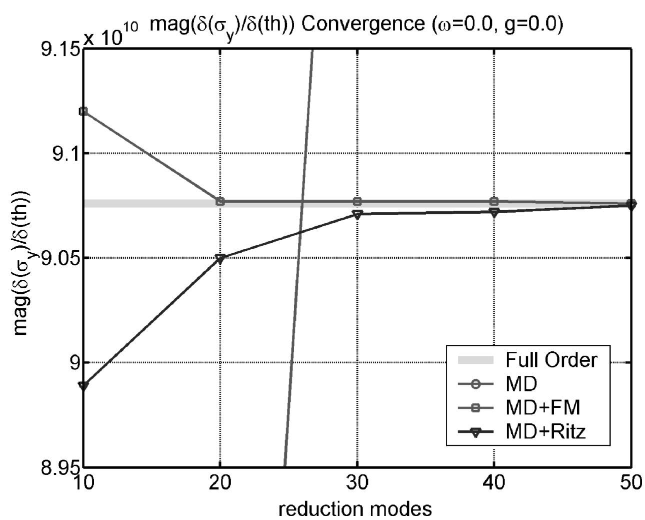

Fig. 9 High AR wing reduced-basis STRESS SENSITIVITY convergence studies (static case, no damping). (Note: Because of large errors the MD curve for the case of natural modes is mostly out of the range shown in the figure.)

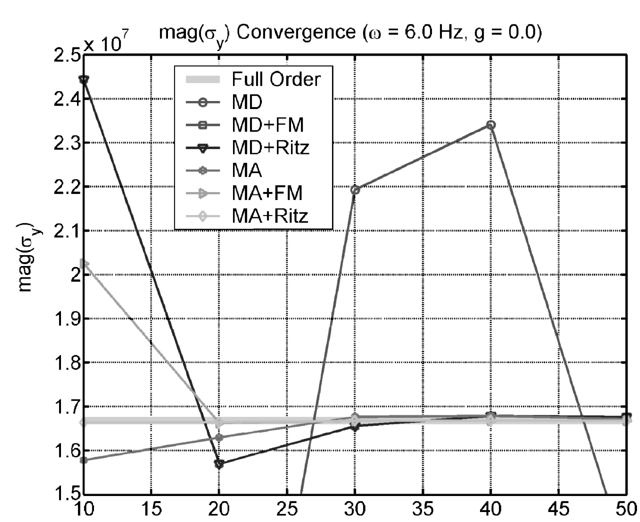

Fig. 10 High AR wing reduced-basis STRESS convergence studies (dynamic case at 6 Hz, no damping). (Note: Because of large errors the MD curve for the case of natural modes is mostly out of the range shown in the figure.)

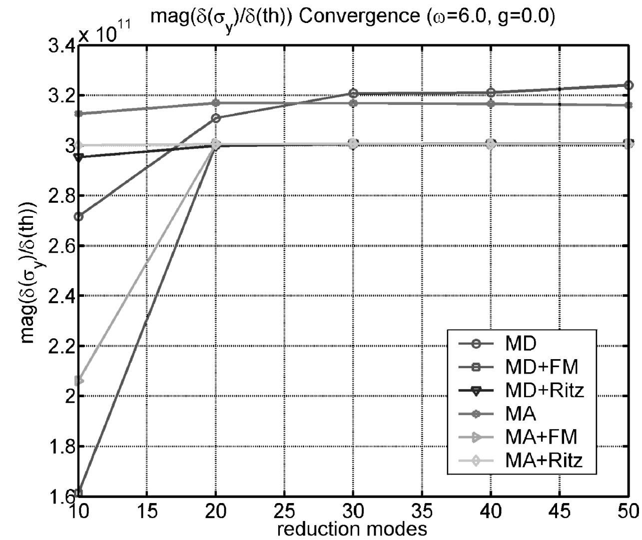

Fig. 11 High AR wing reduced-basis STRESS SENSITIVITY convergence studies (dynamic case at 6 Hz, no damping). (Note: Because of large errors the MD curve for the case of natural modes is mostly out of the range shown in the figure.)

Conclusion

The problem of model order reduction for strain-actuated structures has been presented. Equations of motion have been derived, solution methods have been discussed, and results from test cases have been shown. It has been demonstrated that the local nature of strain actuation has a significant effect on the accuracy of results from reduced-order models when deformations, stresses, and strains, and their sensitivities, are sought. Model order methods presented include MD, MA, Ritz vectors, and FMs. It was pointed out that, because new "smart structures" may contain many actuators activated independently, and many local design variables can vary during optimization, order-reduction techniques may end up with large reduced-order bases.

Appendix

A note is needed regarding the MA formulation of Eq. (19) and the subsequent MA equations in this paper. Actually, in the SOF method—the extension of MA to cases with damping and aerodynamic forces in addition to inertial loads—all forces other than stiffness forces can be moved to the right-hand side of the equations.

Equation 19,

$$((1+jg)[K])\{u(j\omega)\} = \omega^2[M][\Phi]\{q(j\omega)\} + \{F(j\omega)\} + [B]\{v(j\omega)\} \tag{A1}$$

can be actually replaced by

$$[K]\{u(j\omega)\} = \omega^2[M][\Phi]\{q(j\omega)\} - jg[K][\Phi]\{q(j\omega)\} + \{F(j\omega)\} + [B]\{v(j\omega)\} \tag{A2}$$

All MA response and response-sensitivity equations can be based on this formulation.

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